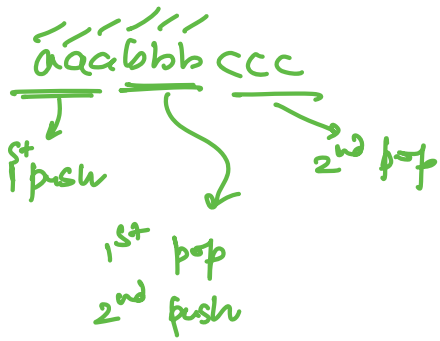
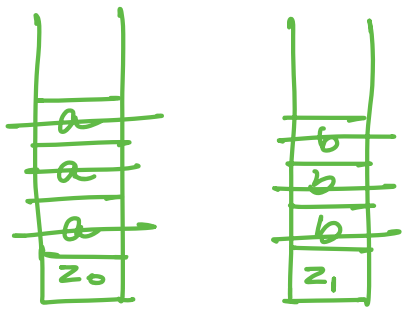
2 Stack PDA

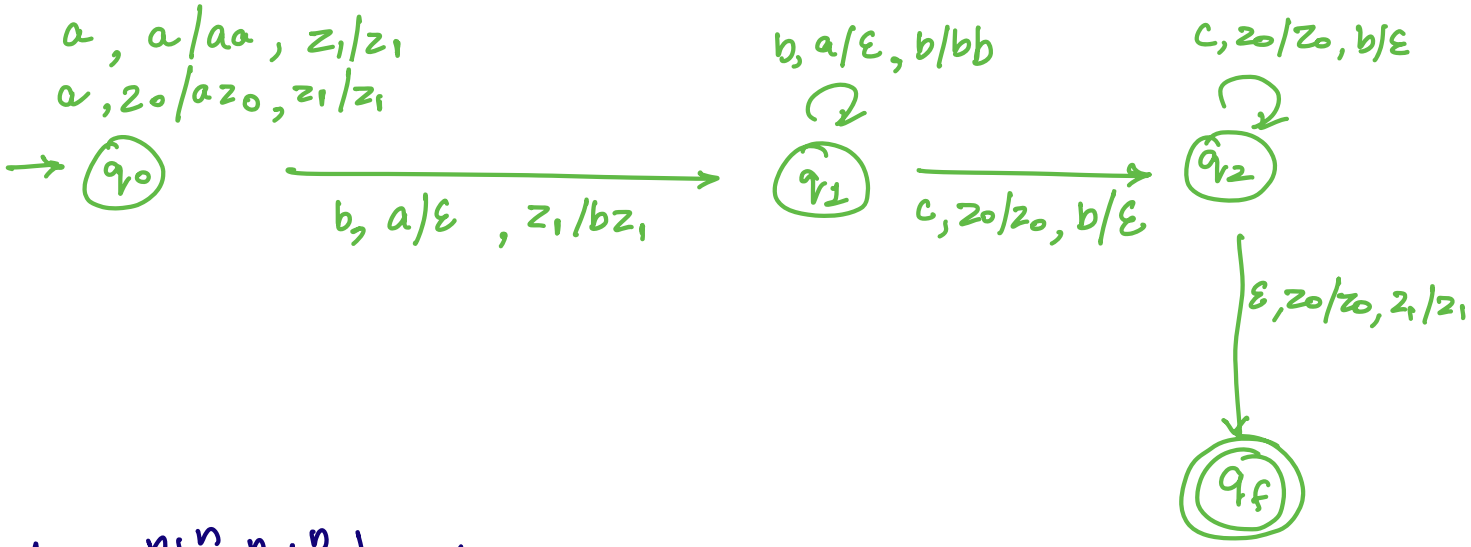
$L = a^n b^n c^n \mid n \geq 1$



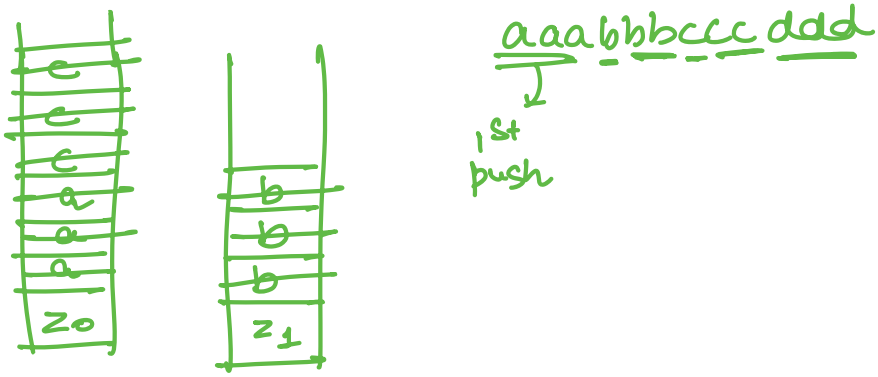


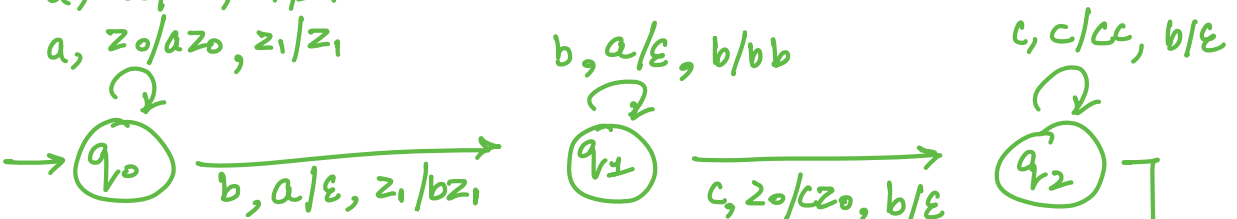
$Q \times \Sigma \cup \{\epsilon\} \times \Gamma \longrightarrow Q \times \Gamma^*$ — 1 Stack PDA

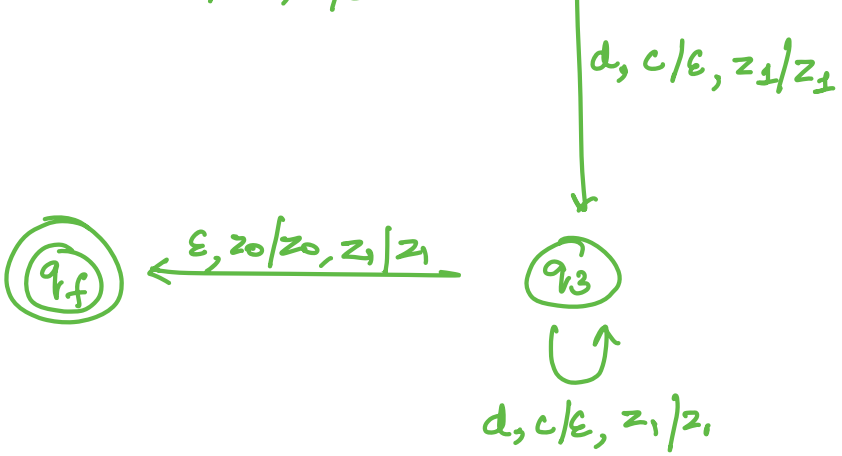
$Q \times \Sigma \cup \{\epsilon\} \times \Gamma \times \Gamma \longrightarrow Q \times \Gamma^* \times \Gamma^*$ — 2 Stack PDA



$L = a^n b^n c^n d^n \mid n \geq 1$



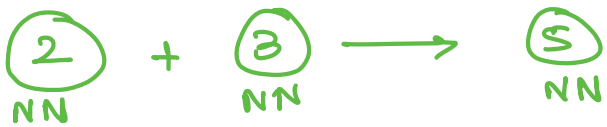




CFL → Single Stack

Properties of CFL:

CFL are closed under union, concatenation, Kleene closure



NN are closed under addition

Union

CFL are closed under union

$L_1 \cup L_2$

CFL CFL

G_1 G_2

$S \rightarrow S_1 | S_2$

G_1 { $S_1 \rightarrow$

G_2 { $S_2 \rightarrow$

New Grammar

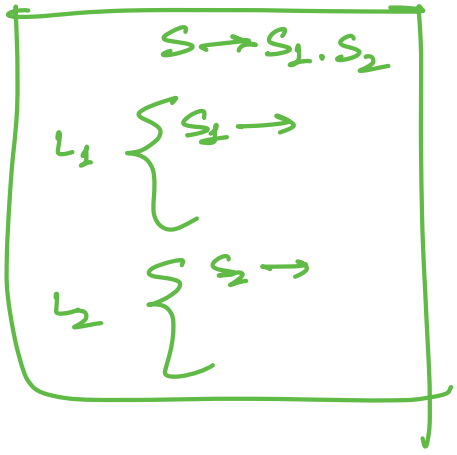
$CFG_1 \cup CFG_2$

Concatenation:

L_1 L_2

$a^n b^n$ $c^m d^m$

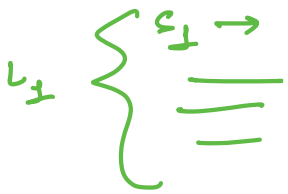
$L_1 . L_2 = a^n b^n c^m d^m$



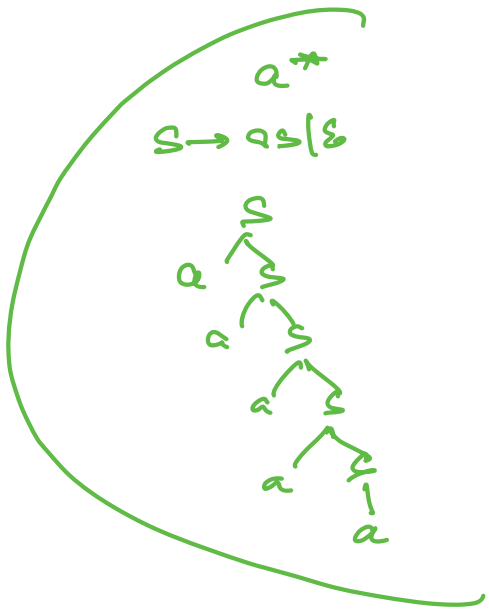
Closure

$L_1 \rightarrow$ cfl

$L_1^* \rightarrow$ cfl



$S \rightarrow S_1 S \mid \varepsilon$



cfl are not closed under intersection & complementation

Intersection:

L_1 L_2

cfl cfl

$L_1 \cap L_2$ cfl?

$L_1 = \{a^n b^n c^m \mid n, m \geq 0\} \rightarrow$ cfl

$L_2 = \{a^m b^n c^n \mid n, m \geq 0\} \rightarrow$ cfl

$L_1 \cap L_2 = \{a^n b^n c^n \mid n \geq 0\}$

↳ not a cfl

cfl are not closed under intersection.

Complementation:

$L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$

Assume: CFL are closed under complementation

$L_1 \rightarrow$ CFL $L_2 \rightarrow$ CFL

$\overline{L_1} \rightarrow$ CFL $\overline{L_2} \rightarrow$ CFL (Assume)

$\overline{L_1} \cup \overline{L_2} \rightarrow$ CFL (Already Proven)

($\overline{L_1}$ CFL, $\overline{L_2}$ CFL) — U closed

$\overline{\overline{L_1} \cup \overline{L_2}} \rightarrow$ CFL (Assume)

$L_1 \cap L_2 \rightarrow$ CFL

↳ we already proved CFL are not closed under intersection

Contradiction
Assumption wrong

CFL are not closed under complementation.

Decidability Problem of CFL:

↳ Write an algo

Membership

String w Language L

$w \in L$?

↓

belong

$L = a^n b^n c^m \mid n, m \geq 1$

$w = aabbc$

CFG → CNF form → $NT \rightarrow NT \cdot NT$

NT → T
ε → ε

n-1 loop

w reach x not in the language

Emptiness

$L = \phi = \{\}$ → No string is accepted

Algo: CFG Simplify → ε production ×
→ unit productions ×
→ useless symbol ×

Start symbol useless ?

Yes → Empty

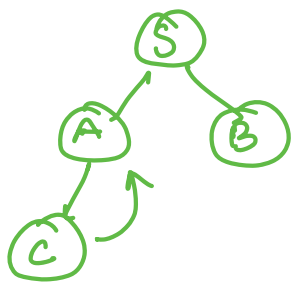
No → Not Empty.

finiteness

CFG → Simplify → ε ×
useless ×
unit ×

$S \to AB$
$A \to ac \mid a$
$C \to aA \mid b$
$B \to a$

Dependency Graph



infinite

Cycle ?

Yes → Infinite

No → finite